INFLUENCE OF EXTERNAL FLOW FIELD ON THE EQUILIBRIUM STATE OF QUASI-GEOSTROPHIC POINT VORTICES

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There have been a number of studies on two-dimensional point vortices, starting from the statistical work by Onsager(1949)[1], who pointed out the existence of negative temperature state. Recently, Yatsuyanagi et al.(2005)[2] performed a large numerical simulation of two-dimensional point vortices, renewing the interest in this classical subject.

Our group investigated the equilibrium state of quasi-geostrophic point vortices in a continuously stratified fluid, both numerically and theoretically.[3, 4] Numerical simulations of mono-disperse vortices ($N = 2000 – 8000$) in an infinite fluid domain were performed using a fast special-purpose computer (MDGRAPE-3) for molecular dynamics simulations. For comparison, the state of maximum Shannon-entropy was determined under the constraints of vertical vorticity distribution, angular momentum and energy. The obtained theoretical distributions agreed well with the numerical results. As the energy increases, the temperature of the equilibrium state is decreased from 'positive', through 'zero inverse' to 'negative'. The zero inverse temperature state is characterized by the Gaussian radial distribution

$$F(r, z) = \frac{P(z)}{\pi} \exp(-r^2).$$

Here, $P(z)$ denotes the vertical distribution. In the positive temperature region (lower energy), the radial distribution is top-hat like in the center region (vertically) and tighter concentrations around the axis of symmetry are found near the upper and lower lids (end-effect). When the energy is higher (in the negative temperature state), vortex distribution shrinks radially in the center region and expands near the lid regions (reversed end-effect).[4]

In this paper, we investigate the influence of external strain and shear on the equilibrium state of quasi-geostrophic point vortices. The objective is to study the internal vorticity distributions of interacting vortex clouds. We approximate the presence of other clouds by locally induced strain and shear fields. At the initial time, the axisymmetric equilibrium state is immersed in the external strain field $U_s = ey, V_s = ex$. In the course of time, the vortex distribution gradually stretches in the $y$-direction and shrinks in the $x$-direction (Fig.1b: top view), which is expected. What is unexpected is the change of the internal vorticity distribution. Figures 1a and b compare the azimuthally averaged radial distributions for a positive temperature state, at the initial time and after $t = 3500$ (averaged from $t = 3500$ to $t = 4000$). The end-effect is weakened substantially and the Gaussian radial distribution is formed at any vertical height, as if it were at zero inverse temperature. The same is found for the state of higher energy, i.e., starting from the negative temperature state. The vertical shear has similar effect on the equilibrium distribution.

References


Figure 1. Vorticity distributions: (a)$t = 0$ (equilibrium), (b)averaged from $t = 3500$ to $t = 4000$ ($\epsilon = 0.08$).